Section 6.4 - The p-series Test (Examples)

Wed
12:30 pm
Nov. 10, 2021

NOTES:

$$P \text{ series} = \sum_{n=1}^{\infty} \frac{1}{n^P} = \frac{1}{1^P} + \frac{1}{2^P} + \frac{1}{3^P} \cdots$$

Ex: $\frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3}$ p series, p=3

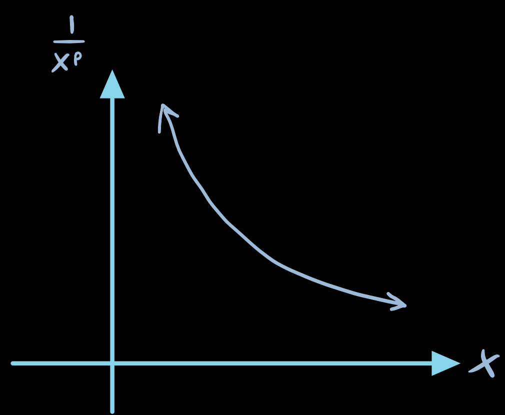
vs

$\frac{1}{1} + \frac{1}{3} + \frac{1}{3^3}$ Geometric Series $r = \frac{1}{3}$

Similar but different

* power stays the same in p-serie

$$f(x) = \frac{1}{x^P}$$



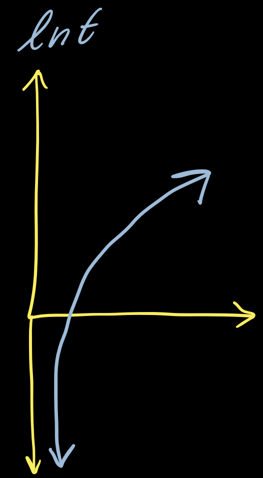
1. ✓
2. ✓
3. ✓

all 3 conditions to apply integral test are satisfied for $x > 0$

$$\int_1^{\infty} \frac{1}{x^P}\,dx \overset{(30a)}{=} \lim_{b\to\infty} \int_1^b x^{-P}\,dx$$

$$= \begin{cases} \lim\limits_{b\to\infty} \left[\dfrac{x^{-P+1}}{-P+1}\right]_1^b & (p \neq 1) \\ \lim\limits_{b\to\infty} \left[\ln x\right]_1^b & (p = 1) \end{cases}$$

$$= \begin{cases} \dfrac{1}{-p+1} \lim\limits_{b\to\infty} \left[b^{-p*1} - 1^{-p*1} \right] \\ \lim\limits_{b\to\infty} [\ln b - \ln 1] \quad (p=1) \end{cases}$$



$$\boxed{= \begin{cases} \text{exists} & p>1 \\ \text{DNE} & p<1 \\ (\text{"="}\infty)\ \text{DNE} & (p=1) \end{cases}}$$ RULE

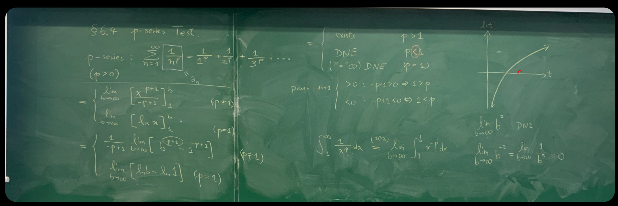
$$\lim_{b\to\infty} b^2 = \text{DNE}$$

$$\lim_{b\to\infty} b^{-2} = \lim_{b\to\infty} \frac{1}{b^2} = 0$$

$$\text{power } -p+1 \begin{cases} >0 : -p+1>0 \Rightarrow 1>p \\ <0 : -p+1<0 = p>1 \end{cases}$$

+ power = DNE

− power = Exists



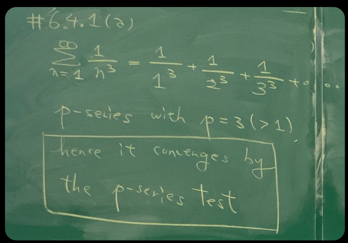
6.4.1.A.

Harmonic Series?

$$\sum_{n=1}^{\infty} \frac{1}{n^3} = \frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} \cdots$$

P-series with $p=3$ (>1)

Hense it converges by p-series test



6.4.2.A.

Approximating $\sum_{n=1}^{\infty} \frac{1}{n^3}$ by using S_4

$$R_4 < \int_4^{\infty} \frac{1}{x^3}\,dx$$

(30a)

$$= \lim \int^{b} x^{-3} \ldots$$

$$\lim_{b\to\infty} \int_4^b x^{-3}\,dx$$

$$= \lim_{b\to\infty} \left[\frac{x^{-2}}{2}\right]_4^b$$

$$= \lim_{b\to\infty} \left[-\frac{1}{2}\left(b^{-2} - 4^{-2}\right)\right]$$

$$= \underbrace{-\frac{1}{2}\lim_{b\to\infty}\left(\frac{1}{b^2} - \frac{1}{4^2}\right)}_{0-\frac{1}{16}} = \frac{1}{32}$$

← small, 4 = good estimate for series!

* he will not ask us to make a judgement call on weather it's good or not

6.4.3.A

We need $R_N < \int_N^{\infty} \frac{1}{x^3}\,dx$

It suffices for us to have

$$\underbrace{\int_N^{\infty} \frac{1}{x^3}\,dx} < E$$

$$\lim_{b\to\infty} \int_n^b x^{-3}\,dx < 5 \times 10^{-5}$$

$$\Rightarrow \lim_{b\to\infty} \left[\frac{x^{-2}}{-2}\right]_N^b < 5 \times 10^{-5}$$

$$= -\frac{1}{2}\left[\underbrace{\lim_{b\to\infty}\frac{1}{b^2}}_{=0} - \frac{1}{N^2}\right] < 5 - 10^{-5}$$

$2 < 5$

$\frac{1}{2} > \frac{1}{5}$

$$2\left(\frac{1}{2N^2} < 5 \times 10^{-5}\right) \Rightarrow \frac{1}{N^2} < 10^{-4}$$

$$\boxed{N^2 > \sqrt{10^4} = N > 100}$$

at least 101 terms!

$\Rightarrow N > \sqrt{\frac{1}{10^{-4}}} = 100$

$\Rightarrow$ $\boxed{\text{Use at least 101 terms}}$

#6.4.3(a)

We need $R_N < E$.

Since $R_N < \int_N^\infty \frac{1}{x^3}\,dx$,

it suffices for us to have

$$\int_N^\infty \frac{1}{x^3}\,dx < E.$$

$\Rightarrow \lim_{b\to\infty} \int_N^b x^{-3}\,dx < 5\times 10^{-5}$

$\Rightarrow \lim_{b\to\infty} \left[\frac{x^{-2}}{-2}\right]_N^b < 5\times 10^{-5}$

$\Rightarrow -\frac{1}{2}\left[\underbrace{\boxed{\lim_{b\to\infty}\frac{1}{b^2}}}_{=0} - \frac{1}{N^2}\right] < 5\times 10^{-5}$

$\Rightarrow 2\left[\frac{1}{2N^2}\right] < \left[5\times 10^{-5}\right]2$

$\Rightarrow \frac{1}{N^2} < 10^{-4}$

$\Rightarrow N^2 > \frac{1}{10^{-4}}$

$2 < 5$

$\frac{1}{2} > \frac{1}{5}$

$\frac{1}{2}x + 4 = 7$